Higher Order QCD Corrections to $b \to c\bar{c}s$

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Abstract

We calculate the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to the decay rate $b \to c\bar{c}s$. For reasonable values of m_c/m_b this term is of the same order as both the one-loop and $\mathcal{O}(\alpha_s^2 \log^2 m_W/m_b)$ corrections to the decay rate. For $m_c/m_b = 0.3$ the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections enhance the rate by $\sim 18\%$. We also discuss the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to R_τ, the B semileptonic branching fraction and the charm multiplicity.

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I. INTRODUCTION

The doubly-charmed decay mode of the B meson, $B \to X_{c\bar{c}s}$, has been the object of recent interest, since this mode makes a significant contribution to the inclusive B semileptonic branching fraction [1,2]. Recently, the one-loop corrections to $b \to c\bar{c}s$ were calculated [3,4] (see also [5]) and found to be substantial, giving a $\sim 22\%$ enhancement to the tree level rate (for $\hat{m}_c = 0.30$). This is significantly larger than the corresponding $\sim 5\%$ $\mathcal{O}(\alpha_s)$ correction to $b \to c\bar{u}d$ decay. When combined with the additional radiative corrections, this brings the theoretical prediction for the semileptonic branching fraction into agreement with the experimental observation, within the theoretical uncertainties [6].

Since the typical energy released in the decay, $\Delta \equiv m_b - 2m_c$ (neglecting the s quark mass), is much less than m_b, one might expect the relevant scale for the perturbative corrections to $b \to c\bar{c}s$ to be significantly less than m_b. Indeed, as stressed in Ref. [7], the energy release in this process is so small that the assumption of local duality may not hold; it has been argued in Ref. [8] that deviations from duality would not show up at any finite order in the operator product expansion. However, even if the assumption of local duality does hold in this instance, this low scale would result in an even greater enhancement of this mode over the tree-level result. This is a higher order effect which requires a full two-loop calculation to address, which we have not attempted. However, in the approach of Brodsky, Lepage and Mackenzie (BLM) [9] useful information may be obtained by simply calculating the n_f dependent piece of the order α_s^2 contribution to the decay. This determines the contribution of $\mathcal{O}(\alpha_s^2 \beta_0)$, where $\beta_0 = 11 - \frac{2}{3} n_f$. Since β_0 is large, this term dominates the two loop result for many processes. The BLM scale μ_{BLM} for the one-loop correction is defined as the scale at which the $\mathcal{O}(\alpha_s^2 \beta_0)$ correction is absorbed in the one-loop correction. This approach has recently been used to estimate the two-loop corrections to semileptonic top, bottom and charm decays [10,11].

In this paper we calculate the $\mathcal{O}(\alpha_s^2 \beta_0)$ correction to the decay $b \to c\bar{c}s$. We will find that this term enhances the decay rate by almost as much as the one loop term, and is of the same

size as the $\mathcal{O}(\alpha_s^2 \log^2 m_W/m_b)$ correction. However, as we will discuss, the $\mathcal{O}(\alpha_s^2 \beta_0)$ term is not necessarily expected to dominate the remaining uncomputed two-loop corrections.

In Section 2 we compute the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to the mode $b \to c\tau\bar{\nu}_\tau$. This contribution arises from strong interaction corrections to the bc vertex, and the result can be related to a piece of the $b \to c\bar{c}s$ correction by taking $m_\tau = m_c$. These corrections are interesting in their own right as they give $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to the ratio

$$R_\tau \equiv \frac{\Gamma(b \to X_c\tau\bar{\nu}_\tau)}{\Gamma(b \to X_c e\bar{\nu}_e)}. \tag{1.1}$$

We will find that the two-loop corrections to this ratio are under control. In Section 3 we calculate the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to the $\bar{c}s$ vertex. We give our conclusions in Section 4.

II. $\mathcal{O}(\alpha_S^2 \, \beta_0)$ CORRECTIONS TO $b \to c\tau\bar{\nu}_\tau$

The rates for $B \to X_c e\bar{\nu}_e$ and $B \to X_c\tau\bar{\nu}_\tau$ may be written as power series in α_s and $\Lambda_{\mathrm{QCD}}/m_b$ [12]. The leading order result in $1/m_b$ reproduces the parton model, while to $\mathcal{O}(1/m_b^2)$ two unknown nonperturbative parameters, $\bar{\Lambda}$ and λ_1, arise. The ratio R_τ defined in Eq. (1.1) provides a potential constraint on these parameters, although the uncertainty in the measurement is currently too large for these constraints to be useful [15–18]. As in the case with massless leptons [11] the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to this process are quite large; however, these corrections largely cancel in the ratio R_τ.

We write the semitauonic decay of a b-quark in terms of the quark pole masses m_b and m_c as

$$\Gamma(b \to c\tau\bar{\nu}_\tau) = \frac{G_F^2 m_b^5}{192\pi^3} \Gamma^{(0)}(\hat{m}_c, \hat{m}_\tau) \left(1 + \frac{\alpha_s(m_b)}{\pi} \Gamma^{(1)}(\hat{m}_c, \hat{m}_\tau) \right. \tag{2.1}$$
$$\left. + \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 \Gamma_\beta^{(2)}(\hat{m}_c, \hat{m}_\tau) + \dots \right)$$

where $\hat{m}_c \equiv m_c/m_b$, $\hat{m}_\tau \equiv m_\tau/m_b$, $\beta_0 = 11 - \frac{2}{3}n_f$ is the QCD β-function, and n_f is the number of light quark flavors running through the vacuum polarization loops. The ellipsis denote terms $\mathcal{O}(\alpha_s^2)$ and higher. The one-loop correction $\Gamma^{(1)}(\hat{m}_c, \hat{m}_\tau)$ is given in Ref. [14].

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To compute the $\mathcal{O}(\alpha_s^2 \beta_0)$ term $\Gamma_\beta^{(2)}$ we follow the work of Smith and Voloshin [10] and compute the $\mathcal{O}(\alpha_s)$ rate with a finite gluon mass, $\Gamma^{(1)}(\hat{m}_g, \hat{m}_c, \hat{m}_\tau)$. The $\mathcal{O}(\alpha_s^2 \beta_0)$ correction in the true theory, $\Gamma_\beta^{(2)}(\hat{m}_c, \hat{m}_\tau)$, can be found from this rate by performing the weighted integral

$$\Gamma_\beta^{(2)}(\hat{m}_c, \hat{m}_\tau) = -\beta_0 \frac{\alpha_s^{(V)}(m_b)}{4\pi} \int_0^\infty \frac{dm_g^2}{m_g^2} \left(\Gamma^{(1)}(\hat{m}_g, \hat{m}_c, \hat{m}_\tau) - \frac{m_b^2}{m_g^2 + m_b^2} \Gamma^{(1)}(\hat{m}_c, \hat{m}_\tau) \right) \quad (2.2)$$

where $\alpha_s^{(V)}(m_b)$ is the strong coupling defined in the V-scheme of Ref. [9], and is related to the coupling α_s defined in the $\overline{MS}$ scheme by

$$\alpha_s^{(V)}(\mu) = \alpha_s(\mu) + \frac{5}{3} \frac{\alpha_s^2(\mu)}{4\pi} \beta_0 + \dots . \quad (2.3)$$

We have obtained a lengthy analytic expression for $d\Gamma(m_g)/dq^2$, where $\sqrt{q^2}$ is the invariant mass of the lepton pair, which we have integrated numerically over q^2 and m_g to obtain $\Gamma_\beta^{(2)}$. Since the results are very sensitive to $\hat{m}_c$ and $\hat{m}_\tau$, we have chosen to follow the approach of Refs. [18] and express these ratios as a power series in $1/m_B$:

$$\hat{m}_c = \frac{\bar{m}_D}{\bar{m}_B} - \frac{\bar{\Lambda}}{m_B} \left(1 - \frac{\bar{m}_D}{\bar{m}_B} \right) - \frac{\bar{\Lambda}^2}{m_B^2} \left(1 - \frac{\bar{m}_D}{\bar{m}_B} \right) + \frac{\lambda_1}{2m_B m_D} \left(1 - \frac{\bar{m}_D^2}{\bar{m}_B^2} + \dots \right) \quad (2.4)$$

$$= 0.372 - 0.628 \frac{\bar{\Lambda}}{m_B} - 0.628 \frac{\bar{\Lambda}^2}{m_B^2} + 1.16 \frac{\lambda_1}{m_B^2}$$

$$\hat{m}_\tau = \frac{m_\tau}{\bar{m}_B} \left(1 + \frac{\bar{\Lambda}}{m_B} + \frac{\bar{\Lambda}^2}{m_B^2} - \frac{\lambda_1}{2m_B^2} + \dots \right)$$

$$= 0.334 + 0.334 \frac{\bar{\Lambda}}{m_B} + 0.334 \frac{\bar{\Lambda}^2}{m_B^2} - 0.167 \frac{\lambda_1}{m_B^2} ,$$

where we have defined the spin-averaged meson masses

$$\bar{m}_D \equiv \frac{m_D + 3m_{D^*}}{4} = m_c + \bar{\Lambda} - \frac{\lambda_1}{2m_D} + \dots \simeq 1975 \, \text{MeV} \quad (2.5)$$

$$\bar{m}_B \equiv \frac{m_B + 3m_{B^*}}{4} = m_b + \bar{\Lambda} - \frac{\lambda_1}{2m_B} + \dots \simeq 5313 \, \text{MeV}.$$

To the order in which we are working we can just use the leading term in our perturbative calculation. We find

$$\Gamma(B \to X_c \tau \bar{\nu}_\tau) = |V_{bc}|^2 \frac{G_F^2 m_B^5}{192\pi^3} [0.082] \left[1 - 1.94 \frac{\bar{\Lambda}}{m_B} - 1.29 \left(\frac{\alpha_s(m_b)}{\pi} \right) \right.$$

$$\left. - 1.28 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 + \mathcal{O} \left(1/m_B^2, \alpha_s/m_B, \alpha_s^2 \right) \right]. \quad (2.6)$$

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For completeness, we also give the result for $\hat{m}_c = 0.3$ and $m_b = 4.80\,\mathrm{GeV}$,

$$\Gamma(B \to X_c \tau \bar{\nu}_\tau) = |V_{bc}|^2\, \frac{G_F^2 m_b^5}{192\pi^3}\, [0.114] \left[1 - 1.39 \left(\frac{\alpha_s(m_b)}{\pi} \right) - 1.58 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 \right.$$
$$\left. + \mathcal{O}\left(1/m_b^2, \alpha_s^2 \right) \right]. \tag{2.7}$$

As is the case for $b \to c e \bar{\nu}_e$ decays, the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections in Eqs. (2.6) and (2.7) are quite large, corresponding to a low BLM scale for this process. However, these corrections largely drop out of the ratio R_τ. Combining Eq. (2.6) with the results of [11], we find

$$R_\tau = 0.224 \left[1 + 0.24 \frac{\alpha_s(m_b)}{\pi} + 0.15 \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \beta_0 - 0.29 \frac{\overline{\Lambda}}{m_B} + \mathcal{O}\left(1/m_B^2, \alpha_s^2 \right) \right] \tag{2.8}$$
$$\simeq 0.224 \left[1 + 0.017 + 0.007 - 0.29 \frac{\overline{\Lambda}}{m_B} + \mathcal{O}\left(1/m_B^2, \alpha_s^2 \right) \right]$$

where we have taken $\alpha_s(m_b) = 0.23$ in the second line. The perturbation series appears well behaved, and the corresponding BLM scale for R_τ is $\mu_{\mathrm{BLM}} = 0.29\, m_b$.

III. $\mathcal{O}(\alpha_S^2 \beta_0)$ CORRECTIONS TO $b \to c\bar{c}s$

Neglecting the s quark mass[1], we write the width for $b \to c\bar{c}s$ decays (where the final state includes an arbitrary number of gluons and light quarks) as

$$\Gamma(b \to c\bar{c}s) = \frac{G_F^2 m_b^5}{64\pi^3} \Gamma^{(0)}(\hat{m}_c) \left(1 + \frac{\alpha_s(m_b)}{\pi} \Gamma^{(1)}(\hat{m}_c) + \left(\frac{\alpha_s(m_b)}{\pi} \right)^2 \Gamma^{(2)}(\hat{m}_c) + \dots \right) \tag{3.1}$$

where

$$\Gamma^{(0)}(x) = \sqrt{1 - 4x^2} \left(1 - 14x^2 - 2x^4 - 12x^6 \right) + 24x^4 \left(1 - x^4 \right) \ln \left(\frac{1 + \sqrt{1 - 4x^2}}{1 - \sqrt{1 - 4x^2}} \right) \tag{3.2}$$

is the tree-level result; $\Gamma^{(0)}(0.30) = 0.196$. The complete one-loop corrections may be obtained from Refs. [3,4]; for $\hat{m}_c = 0.30$ one obtains $\Gamma^{(1)}(0.3) = 2.99$. Taking $\alpha_s(m_b) = 0.23$, this corresponds to a 22% enhancement of the rate over the tree level result.

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[1] Since $m_s \sim \Lambda_{\mathrm{QCD}}$, we will treat terms of order m_s^2 to be of the same size as terms of order Λ_{QCD}^2, which we are neglecting.

Since the four-quark operators responsible for nonleptonic b decays run in the effective theory below m_W the $\mathcal{O}(\alpha_s^2)$ contributions to the decay are more complicated than for semileptonic decays. We write the $\mathcal{O}(\alpha_s^2)$ contribution to the decay rate as

$$\Gamma^{(2)}(\hat{m}_c) = c_1 \ln^2 \frac{m_W}{m_b} + c_2(\hat{m}_c) \ln \frac{m_W}{m_b} + c_3(\hat{m}_c)\beta_0 + c_4(\hat{m}_c) \tag{3.3}$$

where $c_1 = 4$ [19]. The subleading log contribution $c_2(\hat{m}_c)$ was calculated in Ref. [4]; for $\hat{m}_c = 0.30$ these authors find $c_2 = 3.34$.

Clearly the requirement that c_3 dominates the two-loop correction, implicit in the BLM approach, will not hold in this process, since the non-vacuum polarization terms c_1 and c_2 are enhanced by powers of $\ln m_W/m_b$. Separating these terms out, we may instead hope that c_3 dominates over c_4 due to the factor of β_0. However, even this assumption may not hold. Voloshin [3] has shown that in the limit in which the charm is produced nearly at rest c_4 receives a large enhancement. For $b \to c\bar{u}d$, in this limit the analog of c_4 is of order π^2, whereas for $b \to c\bar{c}s$ the Coloumb exchange graphs between the two slowly-moving charmed quarks give a contribution to c_4 of order π^4. While it is not known whether this enhancement is relevant for the physical value of $\hat{m}_c$, it indicates that the $\mathcal{O}(\alpha_s^2\beta_0)$ terms need not dominate over the $\mathcal{O}(\alpha_s^2)$ terms. Nevertheless, as a first step towards understanding the size of the two-loop corrections to this process, we may calculate c_3.

While the complete series of leading and subleading logs has been summed to all orders [19,4], we cannot consistently use these results since we are not summing all terms of $\mathcal{O}(\alpha_s^n \log^{n-2}(m_b/m_W)\beta_0)$. However, as was stressed in Ref. [13], $\ln(m_W/m_b) \approx 2.8$ is not a large number, and the leading log expansion does not seem to work well for nonleptonic b decays. For example, for $b \to c\bar{u}d$ decay the subleading $\mathcal{O}\left(\alpha_s^2 \ln(m_W/m_b)\right)$ term is $2/3$ the size of the leading $\mathcal{O}\left(\alpha_s^2 \ln^2(m_W/m_b)\right)$ term. Therefore, we choose to work consistently to $\mathcal{O}(\alpha_s^2)$ and discard the rest of the leading and subleading log terms. The neglected terms of $\mathcal{O}(\alpha_s^3)$ and above are likely to be much smaller than the uncomputed $\mathcal{O}(\alpha_s^2)$ corrections.

The calculation of c_3 is simplified due to the fact that the graphs factorize into the contribution from the upper bc vertex and the contribution from the lower $\bar{c}s$ vertex. The

upper vertex contribution can be simply obtained from the corrections to $b \to c\tau\overline{\nu}_\tau$ (by making the substitution $m_\tau \to m_c$), while the contribution from the lower vertex require an additional calculation.

For the lower vertex corrections, the kinematic structure of the phase space allows us to express the integrals over the momenta of the $\overline{c}$ quark, s quark, and gluon in terms of the spectral density of the charged V-A current (the imaginary part of the charged current vacuum polarization),

$$\Gamma^{(1)}_{\text{lower}}(m_g) = \int \mathcal{M}_\mu \mathcal{M}^*_\nu \, \text{Im}\, \Pi_{\mu\nu}(q^2) \, \frac{(2\pi)^3}{2m_b} \sqrt{s} \, d\tau_2(P_b; p_c, q) \, dq^2, \tag{3.4}$$

where $\mathcal{M}_\mu$ is the contribution from the bc line and $\text{Im}\,\Pi_{\mu\nu}(q^2)$ is the imaginary part of the vacuum polarization. The tensor structure of the vacuum polarization can be decomposed into a transverse and a longitudinal contribution,

$$\text{Im}\,\Pi_{\mu\nu}(q^2) = \frac{q_\mu q_\nu}{q^2} P_l(q^2) + \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) P_t(q^2). \tag{3.5}$$

Since the functions $P_l(q^2)$ and $P_t(q^2)$ depend only on the scalar q^2, the integration over $d\tau_2(P_b; p_c, q)$ can be carried out analytically with a simple computation,

$$\Gamma^{(1)}_{\text{lower}}(m_g) = \frac{16\pi}{m_b^7} \int \left\{ \left[(m_b^2 - m_c^2)^2 - q^2(m_b^2 + m_c^2) \right] P_l(q^2) + \right.$$
$$\left. \left[(m_b^2 + m_c^2)^2 + q^2(m_b^2 + m_c^2 - 2q^2) \right] P_t(q^2) \right\} \sqrt{\lambda(m_b^2, m_c^2, q^2)} \frac{dq^2}{\sqrt{q^2}} \tag{3.6}$$

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. The resulting expression is quite lengthy and we do not present it here. The functions $P_t(q^2)$ and $P_l(q^2)$ have been previously calculated (for a massless gluon) to $\mathcal{O}(\alpha_s)$ in the context of QCD sum rules [21].

It is then a simple matter to integrate numerically the resulting expression over q^2 and m_g^2 to obtain c_3 as a function of $\hat{m}_c$. At the "reference point" $\hat{m}_c = 0.3$, we find $c_3(0.3) = 3.7$. Using $\alpha_s(m_b) = 0.23$ and $\beta_0 = 9$, this corresponds to an 18% correction to the tree level result, almost as large as the one loop correction. The values of $c_3(\hat{m}_c)$ for a range of values of $\hat{m}_c$ are given in Table I and plotted in Figs. 1 and 2 along with the separate contributions from the upper and lower vertices.

$\hat{m}_c$	$\Gamma^{(1)}(\hat{m}_c)$	$c_3(\hat{m}_c)$	$\frac{\alpha_s(m_b)}{\pi}\Gamma^{(1)}(\hat{m}_c)$	$\left(\frac{\alpha_s(m_b)}{\pi}\right)^2\beta_0 c_3(\hat{m}_c)$
0.20	0.99	0.83	0.07	0.04
0.25	1.91	2.11	0.14	0.10
0.30	2.97	3.67	0.22	0.18
0.35	4.25	5.81	0.31	0.28
0.40	5.85	8.89	0.43	0.43

TABLE I. Numerical values of the one and partial two loop corrections $\Gamma^{(1)}$ and c_3 for $b \to c\bar{c}s$ decay. In the last two columns we have taken $\alpha_s(m_b) = 0.23$ and $\beta_0 = 9$.
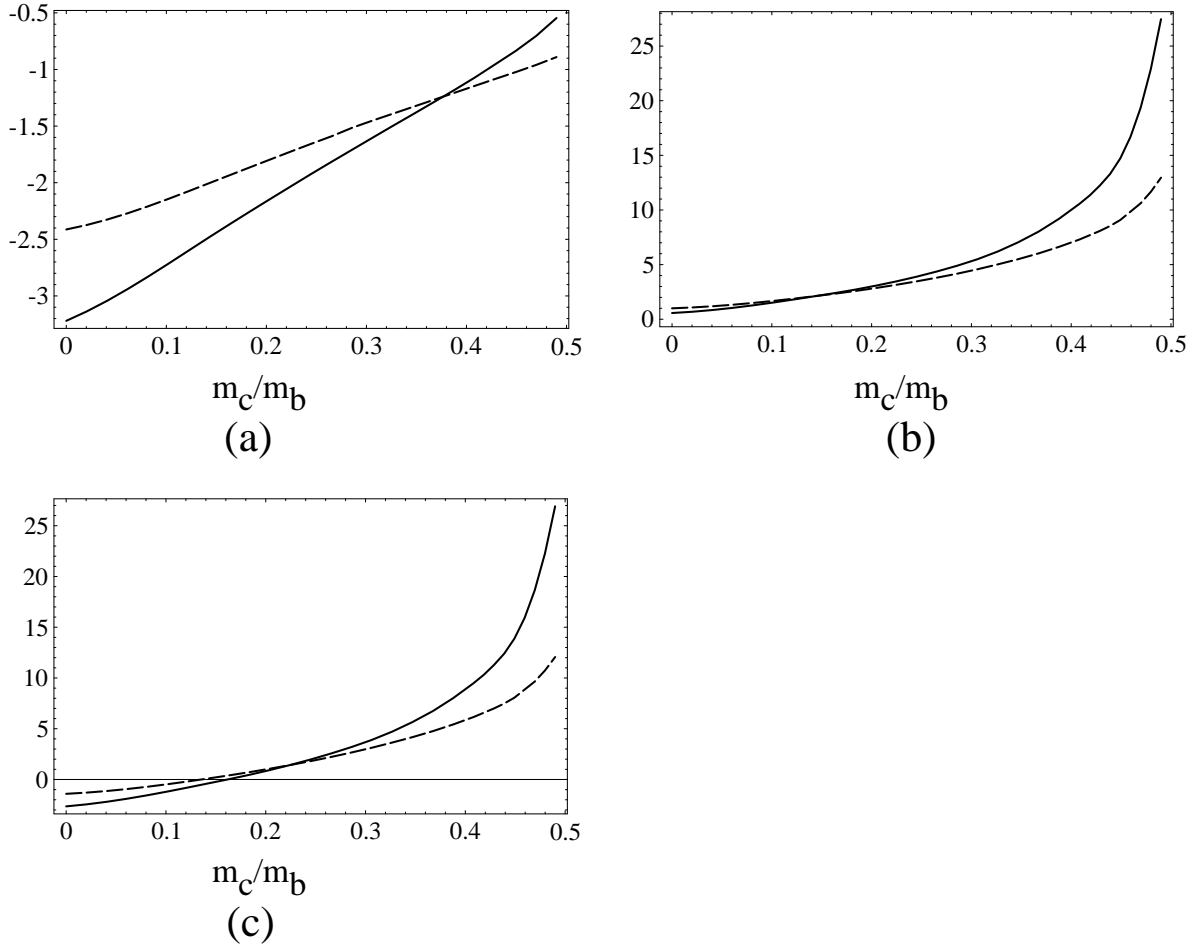

FIG. 1. Contributions to $\Gamma^{(1)}$ (dashed lines) and c_3 (solid lines) from (a) the renormalization of the bc vertex, (b) the renormalization of the $\bar{c}s$ vertex, and (c) the sum of (a) and (b).
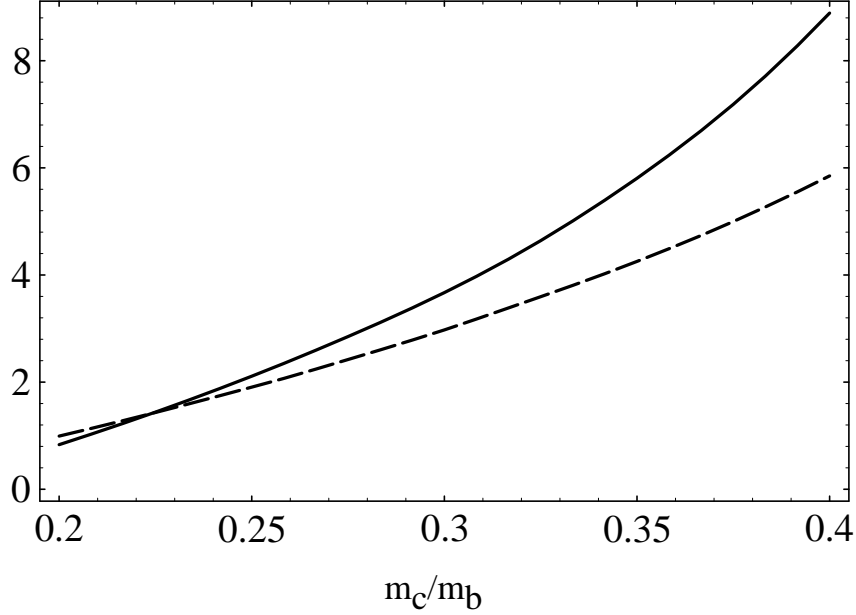
FIG. 2. $\Gamma^{(1)}$ (dashed line) and c_3 (solid line) as functions of $\hat{m}_c$ (expanded view of Figure 1 (c)).

It is useful to compare these results with the leading and subleading log corrections to $\Gamma^{(2)}$. For $\hat{m}_c = 0.3$, these are

$$\left(4\ln^2 \frac{m_W}{m_b} + 3.34\ln \frac{m_W}{m_b} \right) \left(\frac{\alpha_s}{\pi} \right)^2 \simeq (3.5 + 1.0)\beta_0 \left(\frac{\alpha_s}{\pi} \right)^2 \tag{3.7}$$

where we have removed a factor of $\beta_0 = 9$ to allow comparison with the second column in Table I. For $\hat{m}_c = 0.3$ the $\alpha_s^2 \beta_0$ term is roughly the same size as the $\mathcal{O}(\alpha_s^2)$ leading log correction, and a factor of three greater than the $\mathcal{O}(\alpha_s^2)$ subleading log.

Note that the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections to the $\bar{c}s$ vertex are positive for all values of $\hat{m}_c$, while the corrections to the bc vertex are negative. The one loop corrections cancel at $\hat{m}_c \approx 0.14$, while the $\mathcal{O}(\alpha_s^2 \beta_0)$ corrections cancel at a slightly higher value of $\hat{m}_c$. In this situation the BLM scale μ_{BLM} is not physically relevant: at the point where the one loop corrections to the vertices cancel, μ_{BLM} is singular, whereas at the point where the $\mathcal{O}(\alpha_s^2 \beta_0)$ contributions cancel $\mu_{\mathrm{BLM}} = m_b$. In this region the BLM scale for the decay width is unrelated to the BLM scales that would be obtained for the upper and lower vertices individually, and does not reflect the average momentum of the gluons in the diagrams. Therefore we prefer simply to present our results as a contribution to the $\mathcal{O}(\alpha_s^2)$ correction

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to the decay rate.

We also note that since the leading order phase space function $\Gamma^{(0)}(\hat{m}_c)$ is very sensitive to the b and c quark masses, there is a large uncertainty in the total $b \to c\bar{c}s$ width simply due to the uncertainty in the b and c quark masses. Since m_b and m_c are related via the $1/m_Q$ expansion to the corresponding hadron masses, this sensitivity is really an additional hidden source of $1/m_Q$ corrections, just as in the semileptonic decay width. This is made clear if we adopt the approach of the previous section and write $\hat{m}_c$ as a series in $1/m_B$. In this case, the large sensitivity to $\hat{m}_c$ results in a $1/m_B$ correction which is as large as the leading order term,

$$\Gamma(B \to X_{c\bar{c}s}) = |V_{bc}V_{cs}|^2 \, \frac{G_F^2 m_B^5}{64\pi^3} \, [0.057] \left[1 + 9.7\frac{\overline{\Lambda}}{m_B} + \mathcal{O}\left(1/m_B^2, \alpha_s\right) \right]. \tag{3.8}$$

Of course, one could argue that this result is misleading because we are expanding about the extreme value $\hat{m}_c = 0.37$. Nevertheless, the large $1/m_b$ correction shows the sensitivity of the width to the quark masses. Working instead with pole masses and keeping m_b fixed, varying $\hat{m}_c$ between 0.27 and 0.32 results in a factor of two change in the total rate.

It is straightforward to find the $\alpha_s^2 \beta_0$ term for the decay $b \to c\bar{u}d$ from computations of the charmed semileptonic decay [11] and from the results for $R_{e^+e^-}$ [20]. For $\hat{m}_c = 0.3$ this gives

$$\Gamma(b \to c\bar{u}d) = |V_{bc}V_{ud}|^2 \, \frac{G_F^2 m_b^5}{64\pi^3} \, [0.52] \left[1 - 0.67\frac{\alpha_s(m_b)}{\pi} + 4\ln^2\frac{m_W}{m_b}\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \right.$$
$$\left. + 7.17\ln\frac{m_W}{m_b}\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 - 1.11\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \beta_0 + \mathcal{O}\left(\alpha_s^2\right) \right]. \tag{3.9}$$

Combining this with the results of the present work, we find the ratio of the partial widths for $\hat{m}_c = 0.3$,

$$\frac{\Gamma(b \to c\bar{c}s)}{\Gamma(b \to c\bar{u}d)} = 0.376\frac{|V_{cs}|^2}{|V_{ud}|^2} \left[1 + 3.66\frac{\alpha_s(m_b)}{\pi} + 4.80\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \right.$$
$$\left. - 3.83\ln\frac{m_W}{m_b}\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 + \ldots \right] \tag{3.10}$$

The $\alpha_s^2 \beta_0$ correction enhances the tree-level ratio by 22%.

Similarly, the $\mathcal{O}(\alpha_s^2\beta_0)$ enhancement of $\Gamma(b \to c\bar{c}s)$ will decrease the semileptonic branching fraction and increase the charm multiplicity $\langle n_c \rangle$. Combining the result for $b \to c\bar{c}s$ with the $\mathcal{O}(\alpha_s^2\beta_0)$ corrections to the other modes, we find for $\hat{m}_c = 0.3$, an $\mathcal{O}(\alpha_s^2\beta_0)$ correction shift to the semileptonic branching fraction of

$$\delta\left(\frac{\Gamma_{\text{s.l.}}}{\Gamma}\right) = -0.19\,\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 = -0.009. \tag{3.11}$$

The corresponding shift to the charm multiplicity $\langle n_c \rangle$ is

$$\delta\langle n_c \rangle = 0.74\beta_0\left(\frac{\alpha_s(m_b)}{\pi}\right)^2 = 0.036. \tag{3.12}$$

Since we are simply illustrating the effect of the $\mathcal{O}(\alpha_s^2\beta_0)$ terms on these observables, we do not include the remaining perturbative corrections or contributions from rare decay modes in these expressions.

IV. CONCLUSIONS

We have computed the $\mathcal{O}(\alpha_s^2\beta_0)$ contributions to the rate of the nonleptonic decay $b \to c\bar{c}s$ at the parton level. While these corrections do not dominate in any formal limit of the theory, they are a well-defined subset of the complete two-loop corrections. When the perturbation series is expressed in terms of $\alpha_s(m_b)$, the $\mathcal{O}(\alpha_s^2\beta_0)$ corrections are of the same order as both the one-loop corrections and the leading log corrections. For $\hat{m}_c = 0.3$ they provide an additional reduction of $\sim 1\%$ in the semileptonic branching fraction, and increase the charm multiplicity $\langle n_c \rangle$ by ~ 0.04.

These corrections are sufficiently large to cast doubt on the applicability of perturbative QCD to this decay mode. Since there is so little phase space, this is not unexpected. These corrections are in addition to the large $\mathcal{O}(\alpha_s^2)$ corrections suggested by Voloshin [3], as well as the large implicit $1/m_{b,c}$ corrections due to the uncertainties in the c and b masses.

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